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[This question paper contains 4 printed pages.]

Your Roll No. 2022

Sr. No. of Question Paper : 2765 **A**

Unique Paper Code : 62357604

Name of the Paper : Differential Equations

Name of the Course : **B.A. (Prog.) Mathematics : DSE**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** questions are compulsory.

3. Attempt any **two** parts from each question.

1.

a. Solve : 6

$$(8ydx + 8xdy) + x^2y^3(4ydx + 5xdy) = 0.$$

b. Solve : 6

$$(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0.$$

P.T.O.

c. Solve : 6

$$p^2 + 2p\,y\cot(x) - y^2 = 0.$$

d. Show that $1, e^x, e^{2x}$ are the linearly independent solutions of 6

$$y''' - 3y'' + 2y' = 0.$$

What is the general solution? Find the solution y(x) with the property $y(0) = 0$, $y'(0) = 2$, $y''(0) = 3$.

2.

a. Solve : 6.5

$$y' + y\cos(x) = y^2\sin(2x).$$

b. Solve : 6.5

$$y = p^3 + p^2 x.$$

c. Convert into Clairaut's form and hence solve :

$$y = 2xp + xp^2.$$

d. Show that e^{-2x}, xe^{-2x} are the linearly independent solutions of 6.5

$$y'' + 4y' + 4y = 0.$$

What is the general solution? Find the solution y(x) with the property $y(0) = 0$, $y'(0) = 2$.

3.

a. Solve: 6

$$\left(\frac{d^2y}{dx^2}\right) + 2\left(\frac{dy}{dx}\right) + 10y + 37\sin 3x = 0.$$

b. Solve: 6

$$(x^2D^2 + 7xD + 13)y = \log x.$$

c. Apply the method of variation of parameter to solve: 6

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}.$$

d. Find the solution of : 6

$$\left(\frac{d^2y}{dx^2}\right) + 4y = 8\cos 2x$$

given that $y = 0$ and $\frac{dy}{dx} = 0$, when $x = 0$.

4.

a. Solve the following system of equations: 6.5

$$\frac{dx}{dt} - y = t \text{ and } \frac{dy}{dt} + x = 1.$$

b. Solve : 6.5

$$\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2y^2z^2}.$$

c. Solve : 6.5

$$(ydx + xdy)(a - z) + xy\,dz = 0.$$

d. Solve : 6.5

$$\frac{dx}{-xy^2} = \frac{dy}{y^3} = \frac{dz}{axz}.$$

5.

a. Eliminate the arbitrary function f from the equation: 6

$$x + y + z = f(x^2 + y^2 + z^2)$$

to find the corresponding partial differential equation.

b. Find the general solution of the differential equation: 6

$$x^2p + y^2q = (x + y)z.$$

c. Find the integral surface of the linear partial differential equation 6

$$(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$$

which contains the straight line $y = 0$ and $x = 1$.

d. Find the complete integral of the partial differential equation: 6

$$q = (z + px)^2.$$

6.

a. (i) Classify the following partial differential equation into elliptic, parabolic or hyperbolic: 2.5

$$x^2(y - 1)r - x(y^2 - 1)s + y(y - 1)t + xyp - q = 0$$

where $r = \frac{\partial^2 z}{\partial x^2}$, $s = \frac{\partial^2 z}{\partial x\,\partial y}$, $t = \frac{\partial^2 z}{\partial y^2}$, $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.

(ii) Eliminate the arbitrary constants a and b from the equation 4

$$ax^2 + by^2 + z^2 = 1$$

to find the corresponding partial differential equation.

b. Find the general solution of the differential equation: 6.5

$$y^2(x - y)p + x^2(y - x)q = z(x^2 + y^2).$$

c. Show that the following systems of partial differential equations are compatible and hence solve them 6.5

$$p = y\left(1 + \frac{1}{x}\right) + \cos y\,, \quad q = x + \log x - x\sin y\,.$$

P.T.O.

d. Find the complete integral of the partial differential equation: 6.5

$$z = p^2 - q^2.$$